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ABSTRACT

This state-of-the-art review describes the genesis, structure, methods, applications, limitations and goals of the ordering analytic approach to hierarchical analysis, with specific reference to its relevance and use in educational, psychology and behavioral sciences. Hierarchy is defined as an "ordering" of objects such that the relation defined on the objects is reflexive, antisymmetric, transitive and unconnected such as the prerequisite relation. Various empirical and psychometric methodologies for item hierarchies, such as the Guttman and Bart theories, are then outlined. In the documentation of the growth, development and structure of ordering analysis, its prime intent emerges as being the testing of hypothesized item hierarchies and the determination of item hierarchies. Outlines of ordering analysis methods, and search and testing procedures for hierarchies are followed by exemplified applications of ordering analysis. Criticisms of ordering analysis and a recommendation for the need for more research in hierarchical analysis and its impact on psychometrics, measurement, education and behavioral science form the conclusion. (Author/AEP)

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THE ORDERING ANALYTIC APPROACH TO HIERARCHICAL ANALYSIS

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The Ordering Analytic Approach to Hierarchical Analysis

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One comprehensive approach to the psychometric study of test item hierarchies has been determined by ordering analysis and order analysis. This paper will describe the genesis, the structure, the methods, the applications, and the limitations and goals of the ordering analytic approach to hierarchical analysis.

As a preface to this exposition, let me remind everyone how pervasive hierarchies are in psychology and education. Hierarchical organization of psychological phenomena is of interest and importance to many behavioral scientists. In the field of developmental psychology, Piaget (1950) posited that the periods of cognitive development form a specific linear hierarchy. Bart and Smith (1974) suggested that cognitive structures form hierarchies. In the field of cognitive psychology, Neisser and Weene (1962) demonstrated that combinatorial rules such as affirmation and conjunction used to define concepts form a psychological hierarchy. Mandler (1967) stated that the basic memory structures for organizing lists of words were hierarchical, with a limit of about five items at any one level. Collins and Quillian (1969) demonstrated that the attributes of objects tend to form hierarchical structures in memory.

In the field of educational psychology, Gagne (1962, 1968) used task analysis to define hierarchies of learning tasks which were prerequisite to the attainment of terminal objectives. These learning task hierarchies designated the order in which tasks were learned and thus could guide instructional program formation. The research in educational psychology on a priori task

hierarchies has been considerable (e.g., Aivasian, note 1; 1971; Cox & Graham, 1966; Gagne' & Bassler, 1963; Merrill, Burton, & Wood, 1970; White, 1973). These are only a few examples where hierarchies are or have been used to characterize psychological phenomena.

One reason why hierarchies are important in psychology and education is that they designate relationships among phenomena (e.g., if task i is attained, then task j is attained) that are more informative than mere levels of linear association as correlation coefficients provide. Hierarchies designate the order structure among objects. Hierarchies can help to guide practitioners in ordering treatments as in education and they can help to direct researchers in planning sequences of studies and in interrelating the results as in cognitive psychology. Hierarchies also serve as a type of model for scientific explanation.

A Definition of Hierarchy

Any discussant of a basic concept should define the concept. With many concepts, that is an easy task; with the concept of hierarchy, it is not. One reason is the variety of definitions possible. Most people have an intuitive idea of what a hierarchy is and not all of these ideas or approaches are equivalent. So a discussant of the concept hierarchy faces the problem of defining the concept--in our case, in a manner meaningful to behavioral scientists. Unfortunately, this precludes merely stating a mathematical definition which may not have psychological relevance. One task is to develop the definition from a consideration of what is relevant in psychology and then provide an informal definition from those intuitions. This is the path that will be used.

Any hierarchy is an ordering of objects. To further define hierarchy, one must examine the relation that is responsible for the ordering. One relation

that is important in psychology is the prerequisite relation. Two objects are related by the prerequisite relation if the occurrence of one object is prerequisite to the occurrence of the other object. Stating that i is prerequisite to j is equivalent to stating that j implies i , or if j , then i , or i is a necessary but not a sufficient condition for j . Also, i is prerequisite to j asserts that i cannot occur as j occurs. The literature cited thus far is filled with the usage of the prerequisite relation in constructing hierarchies. For example, Piaget (1950) posited that concrete reasoning is prerequisite for formal reasoning in a person but the converse is not the case. Gagne' (1968) contended that the ability to perform certain mathematical skills is prerequisite to the ability to perform other mathematical skills with the converse not always being the case. Other relations in psychology and education have the same logical properties as the prerequisite relation, but, for the purpose of concretely constructing the definition of hierarchy within the context of psychology and education, the prerequisite relation will be emphasized.

The logical properties of the prerequisite relation are rather evident. First, the prerequisite relation is reflexive, because i is prerequisite to itself i . Second, the prerequisite relation is not symmetric, because if i is prerequisite to j , then j is not necessarily prerequisite to i . Third, the prerequisite relation is antisymmetric, because if i is prerequisite to j and if j is prerequisite to i , then i is logically equivalent to j . Fourth, the prerequisite relation is transitive, because if i is prerequisite to j and j is prerequisite to k , then i is prerequisite to k . Fifth, the prerequisite relation is unconnected, because, for any two events (or objects) i and j , it is not necessarily true that either i is prerequisite to j or j is prerequisite to i . Thus, the prerequisite relation is a reflexive, antisymmetric, transitive, and unconnected relation.

Let " $\rightarrow$ " symbolize "is prerequisite to" and " $\leftrightarrow$ " signify "logically equivalent to." Also, let the occurrence of some object, event, or behavior be numbered "1" and the non-occurrence of it with "0." Then graphs and corresponding item response patterns of some hierarchical orderings using the prerequisite relation can be constructed. One such hierarchy is a three-item linear hierarchy which indicates that i is prerequisite to j and j is prerequisite to k which, in turn, would be supported by the occurrence of response patterns (000), (100), (110), and (111) for items i , j , and k respectively. Another hierarchy is a simple three-item branched hierarchy which indicates that i is prerequisite to j and k is prerequisite to j and i is logically independent of k , which, in turn, would be supported by the occurrence of response patterns (000), (100), (001), (101), and (111) for items i , j , and k respectively. Hierarchies can either be linear or non-linear and, as the number of items increases, the number of possible item hierarchies increases also but at a faster rate. Basic then to all hierarchies is that, between any two elements, either a relation of prerequisite, a relation of logical equivalence, or a relation of logical independence holds. The informal definition of hierarchy is then an ordering of objects such that the relation defined on the objects is reflexive, antisymmetric, transitive, and unconnected such as the prerequisite relation. Several approaches have developed for the empirical and psychometric determination of hierarchies.

Methodologies for Hierarchies

The first type of hierarchy to be studied psychometrically was the linear hierarchy. Guttman (1944, 1950) formulated scalogram analysis, a method to determine the linear hierarchy that best fits a set of test items. One way of defining a Guttman scale is the following: a set of test items form a Guttman scale if the summative score of a person for the test items determines exactly what the item responses are for that person. In the case of bivalued items,

Guttman scales form linear hierarchies. As an index of the extent to which a set of items fit a Guttman scale, Guttman formulated the coefficient of reproducibility. Building on Guttman's efforts, Proctor (1970) developed a chi-square statistical procedure to test whether a set of test items form a Guttman scale; his procedure requires that at least three items be considered. Ducamp and Falmagne (1969) extended the mathematical presentation of Guttman theory by extensively articulating the algebraic properties of Guttman scales and other related scales. Though considerable work had been done on Guttman scales, the utility of that work is somewhat limited because most sets of test items do not form linear hierarchies. Bart (1976) demonstrated that linear hierarchies are highly unlikely for more than three items. Though linear hierarchical methods are important, data analytic methods to examine non-linear hierarchies had to be developed--especially, for those interested in learning task hierarchies and cognitive scheme hierarchies.

The Genesis of Ordering Analysis

To assist researchers to investigate non-linear item hierarchies, Airasian (note 1) posed a maximum likelihood procedure based on the multinomial distribution to test the fit of a hypothesized hierarchy for a set of three items. This method, though not generalized to n items, was more general than that of Proctor in that the Proctor method was applicable to linear hierarchies; whereas, the Airasian method was applicable to non-linear hierarchies also. Airasian and Bart (note 2) then introduced tree theory as an alternative measurement model with a boolean algebraic framework. Tree theory was relabelled by Airasian and Bart (note 3) as ordering theory so that the theory would more closely reflect its algebraic framework. Bart and Krus (1973) detailed the first ordering-theoretic method

to determine hierarchies among items and Krus and Bart (1974) delineated an ordering-theoretic method of multidimensional scaling of items; the method described by Krus and Bart became the basis of order analysis. In this paper, the final name change will be instituted as "ordering theory" is renamed "ordering analysis" to parallel more closely the names of other methodologies in psychometrics such as factor analysis. Ordering analysis has as its primary intent either the testing of hypothesized item hierarchies or the determination of item hierarchies.

The Structure of Ordering Analysis

Bart and Krus (1973) articulated the structural basis of ordering analysis:

"In ordering theory (analysis), response patterns for bivalued items are viewed as atoms in a boolean algebra with as many generators as there are items being considered. An ordering (formerly termed a tree) is the union of the obtained atoms and indicates the logical relationships among the items.

Ordering theory (analysis) shares with classical models the item response matrix, but does not use summation across subject rows to express in the form of the correct responses of a subject his standing on the trait measured. This is the first departure of ordering theory (analysis) from older theories of measurement. These models invariably assume that the trait measured is linearly ordered and can be measured with a simple additive model--e.g., a summative score Instead of using summative scores as a starting point for further statistical analysis, ordering theory (analysis) is used to determine logical relationships between items represented in the

item response matrix Ordering theory (analysis) ... employs a logical-mathematical method instead of the otherwise mostly empirical search for relationships between items.

As stated, an ordering is a logical relationship among a set of items. Within that relationship various constituent relationships involving such logical connectives as 'either ... or' and 'and' may be employed. For behavioral researchers the relationship 'is a prerequisite to,' which is the converse of 'implies' or 'if..., then,' is a useful relationship commonly indicated in many theoretical behavior frameworks. ... The salient qualities of the prerequisite relationship for behavioral research warrant its primary consideration in the designation of the logical relationships among items.

One requirement for the prerequisite relationship to be used to inter-relate items is that the items be bivalued--'1' is the item score for item i for subject k if subject k gives a correct response to item i and '0' is the item score for item i for subject k if the subject k gives an incorrect response to item i . Then a defining statement for the prerequisite relationship may be cited ... success on item i is a prerequisite to success on item j if and only if the response pattern (01) for items i and j respectively does not occur. The response patterns (00), (10), and (11) are called confirmatory and the response pattern (01) is called disconfirmatory with respect to the two-item ordering, item i is a prerequisite to item j ."

Ordering analysis can deal with linear and non-linear hierarchies and thus is more general than Guttman scalogram analysis. However, it does have a more restrictive condition for usage than scalogram analysis. With ordering analysis, all items must be bivalued; whereas, polyvalued items are permissible with the scalogram analysis. Also, all subjects must respond to all test items in order for ordering analysis methods to be used.

Though ordering analysis was initially formulated as a boolean algebraic measurement model having the potential to reveal the logical structure among a set of items without regard to how complicated the inter-item logical relationships may be, it has thus far been developed solely for hierarchies involving the prerequisite relation. Ideally, item response patterns should be wholistically analyzed to determine test item hierarchies. No such analysis has yet been developed by ordering analysts. Rather, two-item prerequisite relations are identified and then composed to form the whole test item hierarchy.

Methods of Ordering Analysis

Search Procedures for Hierarchies

Like scalogram analysis, ordering theory is basically a deterministic measurement model. Thus, the problem of accounting for errant response patterns is a difficult one. In ordering analysis, success on item i is a prerequisite to success on item j if the frequency of 01 response patterns for items i and j respectively is less than some pre-conceived level L . This level is the product of some pre-conceived percentage of error to be tolerated (e.g., 0%, 1%, or 5%) and the number of subjects tested. For example, if there are 100 subjects and one allows 2% of the responses to be error, then $L = .02 \times 100 = 2$. The smaller the percentage of error established, the more severe is the analysis and the smaller is the number of two-item prerequisite relations to be identified.

Ordering analysis has two methods of handling error. In one method (Bart and Krus, 1973) some relatively small value of L is established. Those pairs of items having prerequisite relations are then located and combined to form the test item hierarchy. Though this method is manageable and is in computer program form, it can generate intransitivities. In other words, if L is greater than 0, then items i , j , and k may be found such that i is prerequisite to j , j is

prerequisite to k, but i is not prerequisite to k. Such a situation violates the transitivity property of a hierarchy.

In the second method, item response patterns which occur too infrequently are viewed as error and are deleted from further analysis. This preferred ordering analytic method that searches for item hierarchies was discussed by Bart (1978). A group of subjects are given a set of dichotomously-scored test items, and the test item data is scored and stored in a subject X item response matrix. The first step in the procedure is that item response patterns that may be due to error are deleted from the item response matrix. This is done by deleting those item response patterns whose frequencies are less than or equal to some minimum L' such as 1 or 2 and whose combined frequencies are less than or equal to some pre-established percentage of the sample size such as 1 or 2%. This step is based on the contention that item response patterns that actually constitute the true item hierarchy in a test will tend to have higher frequencies of occurrence than item response patterns that do not and thus occur as error item response patterns. The second step after infrequently-occurring item response patterns are deleted involves the derivation of all the inter-item prerequisite relations from the remaining item response patterns with the use of the following rule. Item i is prerequisite to item j if and only if the frequency of 01 item response patterns for items i and j respectively is equal to 0. Thus, from an examination of the 01 item response pattern, frequencies for all possible pairs of items in the test, all the inter-item prerequisite relations can be identified and the item hierarchy can be constructed. This two-step method allows no item intransitivities (i.e., situation in which item i is prerequisite to item j and item j is prerequisite to item k, but item i is not prerequisite to item k) which is incompatible to the nature of a hierarchy and this method incorporates no infrequently-occurring response patterns which may be

attributable to error. Refinements and improvements in ordering-~~theoretical~~^{analytic} search procedures for hierarchies are expected, but this two-step method is presently a preferred search procedure for item and skill hierarchies.

Another search method was developed by van Leeuwe (1974). His method, termed item tree analysis, is derived from ordering ~~theory~~^{analysis} and graph theory and provides a measure of how good a hierarchy fits a set of test items, termed the coefficient of partial reproducibility. However, van Leeuwe's method also needs probabilistic foundations. A third method to identify item hierarchies has been that of Cliff (1975), but is applicable at this ^{time} only to interactive computer testing situations. In another paper, Cliff (1977) has formulated some psychometric indices for item data fitting a hierarchical structure; one of the indices provides a measure of internal consistency which is algebraically related to the classical index of internal consistency KR-20. However, Cliff's indices do not have probabilistic foundations, so that significance tests for values of the indices are not now available. Most search procedures to identify item hierarchies emanate from deterministic frameworks.

Testing Procedures for Hierarchies

The first important procedure in testing for hypothesized hierarchies was that of Proctor (1970) who developed a chi-square statistical method to test whether a set of test items form a Guttman scale; his procedure requires that at least three items be considered. Murray (note 5) refined Proctor's method and developed a rigorous maximum-likelihood procedure to assess the sequential nature of not just item and task data, but also developmental phenomena. Weaker methods have been developed by Parker, et al. (note 6) and Heiner, et al. (note 4). White and Clark (1973) developed an interesting statistical procedure to test whether one skill is a precondition to another skill but due to its complexity

it is doubtful whether the method will be extended to testing hypothesized skill hierarchies involving more than three items.

Wood (note 7) developed a significance test of the number of two-item prerequisite relations which are observed by chance given different tolerance levels of errant response patterns, number of items, and item difficulties. Dayton and Macready (1976) have continued the effort of Murray and produced a general probabilistic model that does not put restrictions on pattern vector probabilities when arbitrary hierarchies are being examined. However, their procedure is not able to handle the full-range of non-linear hierarchies.

Baker and Hubert (1977) formulated an interesting statistical test which assesses the degree to which a hierarchy derived from observed data through the use of an ordering-theoretic search procedure fits a hypothesized or another empirical hierarchy among the same n skills or items. Their non-parametric permutation test provides a conservative test of hierarchy similarity and has not been rigorously assessed either empirically or analytically. Although it is a promising procedure, it has certain weaknesses. For example, it does not take into account differences in the number of respondents determining the hierarchies which may be compared.

Applications of Ordering Analysis

Subsequent to Guttman's initial efforts on hierarchies, research on hierarchies has been addressed mainly either to the search for a best-fitting hierarchy or to the testing of a hypothesized hierarchy among items. Prominent among search approaches is ordering analysis which was formulated to determine even non-linear hierarchies. The foundations of ordering analysis have been described in several papers (Airasian and Bart, 1973; Bart and Krus, 1973; Krus and Bart, 1974; Krus, Bart, and Airasian, 1975). Other research reports applications

of ordering analysis in the investigation of Piagetian task hierarchies (Bart and Airasian, 1974; Bart and Mertens, 1979; Bart, Frey, and Baxter, 1979), propositional logic items (Airasian, Bart, and Greaney, 1975), instructional sequences (Airasian and Bart, 1975), and attitudes toward school (Airasian, Madaus, and Woods, 1975). In one interesting study, Hurst (1980) used ordering analysis to investigate the hierarchical relationships between behaviors in the cognitive and affective domains. Her study was one of the first to attempt to interrelate two item hierarchies.

Items in a test have various types of inter-item relationships which determine different types of item structures. For example, inter-item correlations (e.g., phi coefficients) provide measures of linear relationships between items and determine an item factor structure which indicates to what extent the items measure one underlying trait, or several underlying traits. Other types of inter-item relationships are logical relationships such as the prerequisite relation (i.e., success on item i is prerequisite to success on item j), the logical equivalence relation (i.e., success on item i is prerequisite to success on item j and vice versa), and the independence relation (i.e., performance on item i is independent of performance on item j). Inter-item correlations and then item factor structure can be generated in the examination of linear relationships among items and inter-item prerequisite relations and then an item hierarchy can be generated in the examination of logical relationships among items.

The relationship between hierarchical structure and factor structure has been studied only in one study by Bart (1978). To study the question "Can the factor structure of a test tell us something about the hierarchical structure?", Bart took published item response data for two groups of five dichotomously-scored Law School Admissions Test items such that each group of item data was based on a sample of 1000 subjects and such that one group of items conformed to

a bifactor structure and the other to a unifactor structure. Using a method of data analysis derived from ordering theory which took into account that certain frequencies of response patterns may be due to error, it was determined that there were no inter-item prerequisite relations and thus no item hierarchy in either item data set. This study established that the item factor structure of a test does not necessarily indicate anything about the item hierarchy in a test. The alternative issue as to whether test hierarchical structure indicates anything about test factor structure still remains to be investigated in the examination of the relationship between the two test structures. Thus factor analysis of items cannot be used to determine the item hierarchy in a test and thus cannot be used to examine the optimal instructional sequence among learning experiences or the logical hierarchical structure among items, skills, or educational objectives. A different methodology is required such as ordering analysis which developed through the efforts of Airasian, Bart, Krus, and others, and which allows the search for item hierarchies and the testing of hypothesized item hierarchies to a moderate level of rigor.

As an example of an ordering-analytic application, let me display a hierarchy among six Inhelder-Piaget formal reasoning tasks which were administered to 285 adolescents in the Chicago Public High School System. Each subject received a score of "1" for a task performance that was formal operational in nature and a score of "0" otherwise. Figure 1 provides the hierarchical structure for the six tasks.

Enter Figure 1 about here

From this analysis, it was found that the six tasks are rather tightly inter-related in terms of hierarchical relationships as, for example, the isolation of

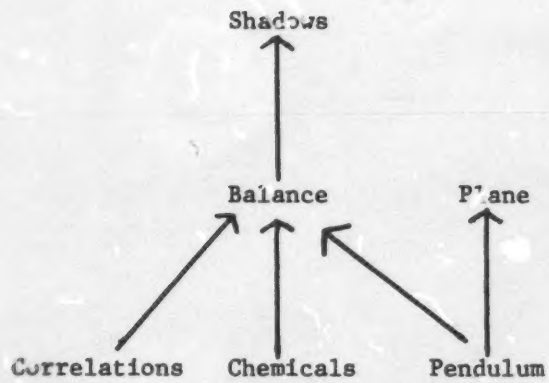


Figure 1. The Hierarchical Structure among Six Formal Operations Tasks.

variables as assessed by the pendulum task was found to be a prerequisite to the scheme of proportion as assessed by the balance and shadows tasks and to the concept of conservation of motion on a horizontal plane as assessed by the plane task. In fact, the hierarchy for the six tasks depicted in Figure 1 had seven inter-task prerequisite relationships. This hierarchy could be consulted by researchers interested in instructional programs directed at effecting formal reasoning skills. Thus, it would be suggested that instruction leading to the attainment of the capacity to isolate variables precede instruction leading to the attainment of the scheme of proportion.

Limitations and Goals of Ordering Analysis

Recently, two criticisms of ordering analytic methods have been made. In a theoretical paper, Bergan (1980) contended that the description of prerequisite relations among items can lead to a loss of information. He suggested that "connections involving direct influence that were not prerequisite in character" should also be considered. My response is that causal relations, prerequisite relations and linear relations among items are clearly meaningful and definable to me; the direct influences Bergan discussed are not evidently meaningful at all. Ordering analysis will continue to emphasize the study of logical relationships among items.

The second paper by Price, Dayton, and Macready (1980) provides empirical evidence that argues for the psychometric superiority of hierarchy search procedures developed by the authors themselves. I anticipate additional studies on this topic because of the limited nature of this study. For example, Cohen's kappa (Cohen, 1960) was used to index the degree of agreement between the "true" generated hierarchy and the hierarchy "discovered" by a given algorithm. No reasons were given why the statistically rigorous choice of the Baker-Hubert lambda (Baker & Hubert, 1977) was not used.

If ordering analysis is to continue to grow, as it should, it must assimilate and/or respond productively to other enterprising approaches to hierarchical analysis such as graph-theoretic ones, log-linear modeling, prediction analysis, latent class models, and path analysis. The full impact of hierarchical analysis on psychometrics, measurement, education, and the behavioral sciences has not yet occurred at all.

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